

+0.1
to fin
3.5.25

+0.1 for A +0.2 ~~for~~



Massachusetts Institute
of Technology (MIT)



Lecture by Pr. Bob Gallagher Aristotle (384–
322 BC), Boole (1815-1864) & Shannon (1916-2001)

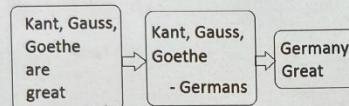
+0.1 6.6.25



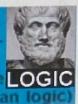
A little
nationalistic,
but this is an
example of
right logic



Bad logic
(abuse
of logic)

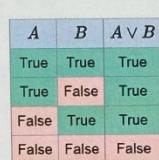


George Boole (1815-1864) developed
The principles of logical thinking have been under-
stood (and occasionally used) since the Hellenic era.
Boole's contribution was to show how to systemize
these principles and express them in equations (Boolean logic)

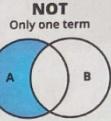
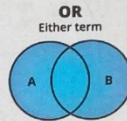
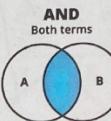


Logical addition (disjunction)

A	B	$F = A \vee B$
0	0	0
0	1	1
1	0	1
1	1	1



BOOLEAN LOGIC



Claude Shannon (1916-2001)
showed how to use Boolean algebra
as the basic for switching technology

George Boole (1815-1864) developed Boolean logic

The principles of logical thinking have been understood (and occasionally used) since the Hellenic era.

Boole's contribution was to show how to systemize these principles and express them in equations (called Boolean logic or Boolean algebra).

Claude Shannon (1916-2001) showed how to use Boolean algebra as the basis for switching technology. This contribution systemized logical thinking for computer and communication systems, both for the design and programming of the systems and their applications.

Logic continues to be abused in politics, religion, and most non-scientific areas.

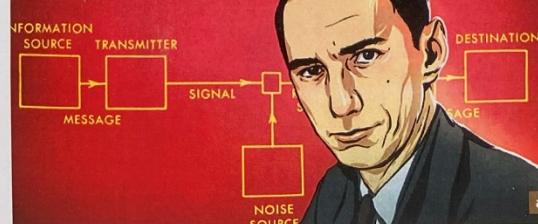
Logic continues to be
abused in politics,
religion and most non-
scientific areas

Resume of Lecture by Pr. Bob Gallagher from MIT

MIT Massachusetts Institute
of Technology (MIT)



The Mathematical Theory of Communication



Creating a reliable
connection over an
unreliable (noisy)
channel
that's
what
IT
is
about
and that's what Shannon did



Sapere aude!

Boolean functions.

$$\bullet \text{NOT}(x) = \begin{cases} 1 & \text{if } x \text{ is } 0 \\ 0 & \text{if } x \text{ is } 1 \end{cases}$$

$$\bullet \text{AND}(x, y) = \begin{cases} 1 & \text{if both } x \text{ and } y \text{ are } 1 \\ 0 & \text{otherwise} \end{cases}$$

$$\bullet \text{OR}(x, y) = \begin{cases} 1 & \text{if either } x \text{ or } y \text{ (or both) is } 1 \\ 0 & \text{otherwise} \end{cases}$$

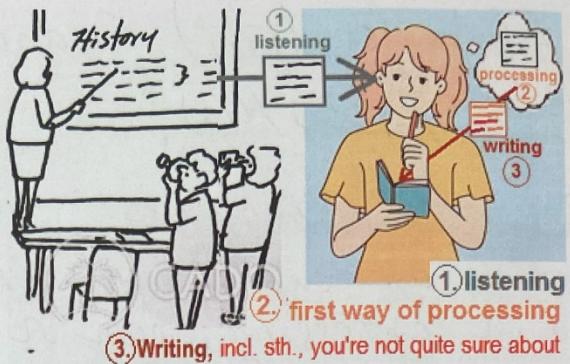
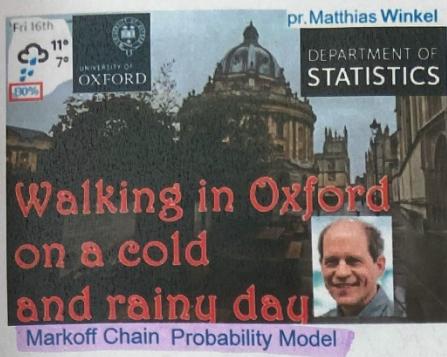
$$\bullet \text{XOR}(x, y) = \begin{cases} 1 & \text{if } x \text{ and } y \text{ are different} \\ 0 & \text{otherwise} \end{cases}$$

NOT	AND	OR
x/x'	$x y/x y$	$x y/x+y$
0/1	0 0 0	0 0 0
1/0	0 1 0	0 1 1
	1 0 0	1 0 1
	1 1 1	1 1 1

XOR

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

d:\ed\IT\Projects\01



CHALK + TALK ink + think
take notes on the lecture yourself

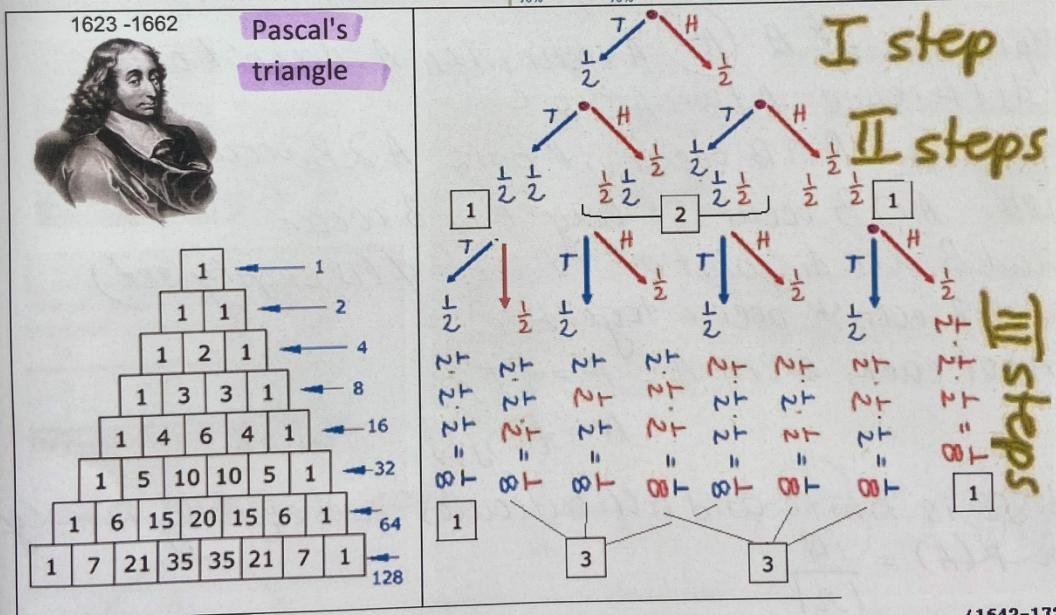
Motivation: 80% chance of rain

Let A_j be the event of rain at 9am on day j of this term, $1 \leq j \leq n$

Suppose the events A_i each have probability P , independently.



School =formalism=> $E=mc^2$ Uni



$$(a+b)^0 =$$

$$(a+b)^1 =$$

$$(a+b)^2 =$$

$$(a+b)^3 =$$

$$(a+b)^4 =$$

$$(a+b)^5 =$$

1

$a+b$

$$a^2 + 2ab + b^2$$

$$a^3 + 3a^2b + 3ab^2 + b^3$$

$$a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

$$a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$$

Newton's Binomial

$$(1+x)^p = \dots$$



$$H = \sum_{x=1}^n p(x) \log_2 \left(\frac{1}{p(x)} \right)$$

$I(x) = \log_2 \left(\frac{1}{p(x)} \right)$ Quantifying information

$$H(x) = \sum_{x=1}^n p(x) H(x)$$

Events & probabilities

Consider an "experiment" which has a set of Ω of $w \in \Omega$. a) Tossing a coin $\Omega = \{H, T\}$ - a coin comes up $A = \{T\}$
 b) throwing a dice - b) $A = \{(3, 6)\}$ - event

$$\Omega = \{(i, j), i, j \in [1, 2, 3, 4, 5, 6]\}$$

set of possible outcomes

A subject of Ω is called event

$w \in \Omega$ is the outcome, we say that A occur if $w \in A$

Complement of A : (A^c) \bar{A} occur when A doesn't occur

Set difference: $A \setminus B = A \cap B^c$

Intersection: $A \cap B$ occur if both A & B occur

Union: $A \cup B$ occur if any A or B occur

A and B are disjoint if $A \cap B = \emptyset$ (is empty set)

A and B can't occur together.

1) $P(A)$ of each event A $A = \{T\}$

$$A = \{(i, j)\}$$

Case: Ω is finite and all outcomes are equally likely

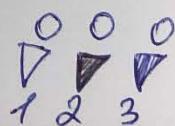
$$\text{then } P(A) = \frac{|A|}{|\Omega|}$$

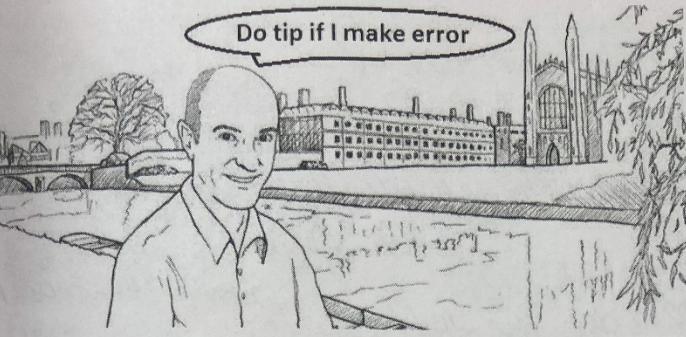
a) coin $|\Omega| = 2$
 $|A| = 1 \quad P(A) = \frac{1}{2}$

b) $|\Omega| = 6$
 $|A| = 1 \quad P(A) = \frac{1}{6}$

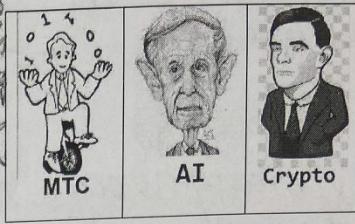
Elementary combinatorics. Arrangement of distinct objects.

Suppose we have n dist-e objects

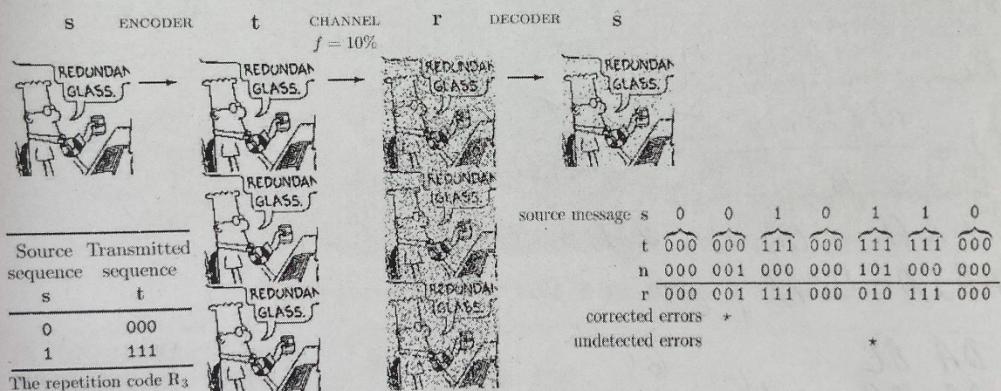
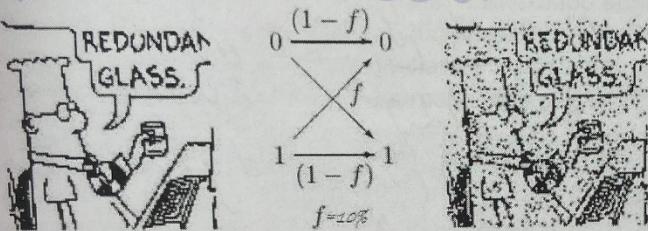
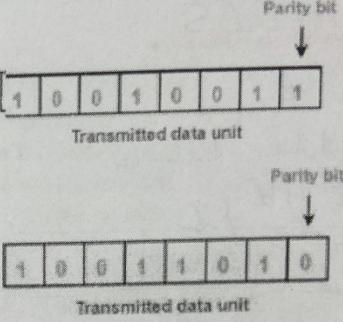
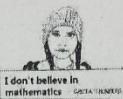




Sir Dr. D. MacKay,
University of Cambridge
(22 April 1967 – 14 April 2016)

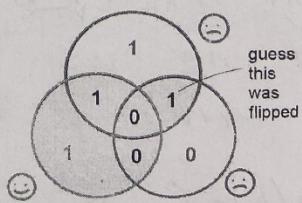
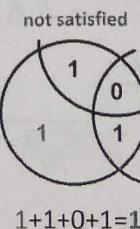
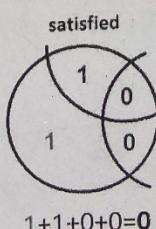
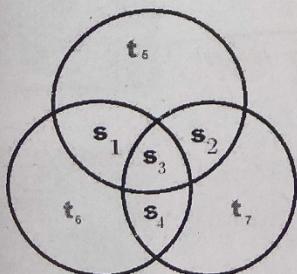
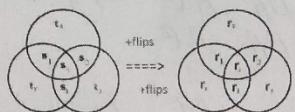


"I believe in clean energy,
but I also believe in mathematics"



7.4. Hamming code.

$$\frac{4}{\sum} \rightarrow \frac{7}{t}$$



How many ways are there to order them

$$3! = 1 \cdot 2 \cdot 3 = 6$$

For n there are n options for 1st \checkmark

then $n-1$ for the 2nd object \checkmark

inductively $n-(n-1)$ -with in all this are

$$n(n-1)(n-2) \dots n \cdot 1$$

How many ways to order the letters of GALOIS

$$n! = 6!$$

UUU UUU

and
adndfz

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{A_n^k}{P_k}$$

$$A = (n-k)$$

$$P(A) = \frac{m_1! \cdot m_2! \cdots m_r!}{n!} \quad m_1 + m_2 + \dots + m_r = n$$

$$n! = n(n-1)(n-2) \dots 2 \cdot 1$$

A₁ A₂ A₃ B C D

$$BA DA AC \quad \downarrow 6!$$

$$BA DA AC \rightarrow l! \quad \frac{6!}{3!}$$

$$L \quad M \quad T$$

$$\underbrace{x_1 x_1}_{m_1}, \underbrace{x_1 x_k}_{m_2}, \underbrace{x_k x_k}_{m_r} \dots$$

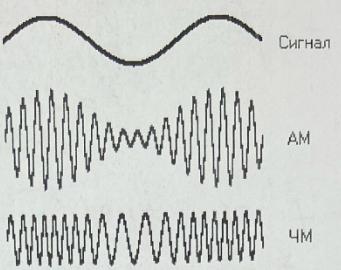
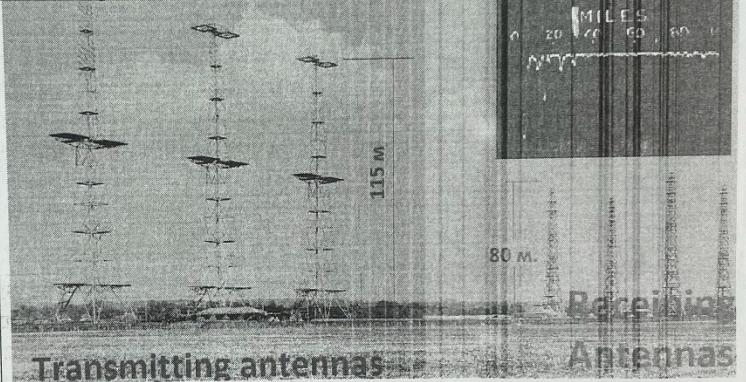
$$ABBAT \rightarrow \frac{5!}{2! 2! l!}$$

$$n = m_1 + m_2 + \dots + m_r$$

$$n! = m_1! m_2! \dots m_r!$$

$$P_n(k) = \binom{n}{k} \cdot P^k \cdot q^{n-k} \quad (1-q)$$

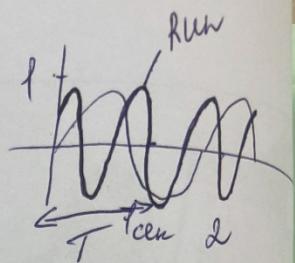
*7.1-
Larionov*

 Reginald A. Fessenden (October 6, 1866 – July 22, 1932)	 Сигнал AM ЧМ	(October 6, 1866 – July 22, 1932) first transmission of speech by radio (1900), and the first two-way radiotelegraphic communication across the Atlantic Ocean (1906)
<p>"Ни одна организация, занимающаяся какой-либо конкретной областью деятельности, никогда не изобретает какие-либо важные разработки в этой области или не внедряет какие-либо важные разработки в этой области до тех пор, пока она не будет вынуждена сделать это из-за внешней конкуренции.." Oxford University Press. The Quarterly Journal of Economics , Feb., 1926, p. 262.</p>		
<i>89! 4 dy</i> Battle of Britain (3 month 3 weeks) 10.07-31.10.1940 	 Transmitting antennas Receiving Antennas	
Radar played a major role in the Battle of England		
H. Nyquist 	$W = K \log m$	
Where W is the speed of transmission of intelligence, m is the number of current values, and, K is a constant.		
	$H = n \log s$ $= \log s^n.$	

$$F_s \geq 2F_{\max} \quad (T_s \leq T_{\text{run}}/2)$$

$$\left. \begin{array}{l} f=1 \quad T=1 \\ f=2 \quad T=\frac{1}{2} \\ f=3 \quad T=\frac{1}{3} \end{array} \right\} T = \frac{1}{f}$$

$$\begin{aligned} f &= 1 \text{ Hz} \\ T &= 1 \text{ sek.} \\ \text{Run} & 120 \\ f &= 2 \text{ Hz} \\ T &= 0,5 \text{ sek} \end{aligned}$$



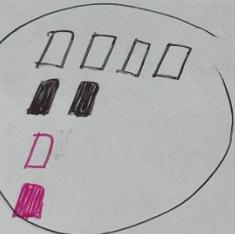
4 - -	+++ +	$\frac{4}{8}$	- 50%
5 - 6	♠ ♠	$\frac{2}{8} \rightarrow \frac{1}{4}$	$\rightarrow 25\%$
7	★	$\frac{1}{8}$	- 12,5%
8	♥	$\frac{1}{8}$	- 12,5%

$\log_2(4) = 2$ compoza

$0,5 \cdot 1$
 $0,125 \cdot 2$
 $0,125 \cdot 3$
 $0,125 \cdot 3$

$0,5$
 $0,5$
 $0,125 \cdot 6$
 $0,45$

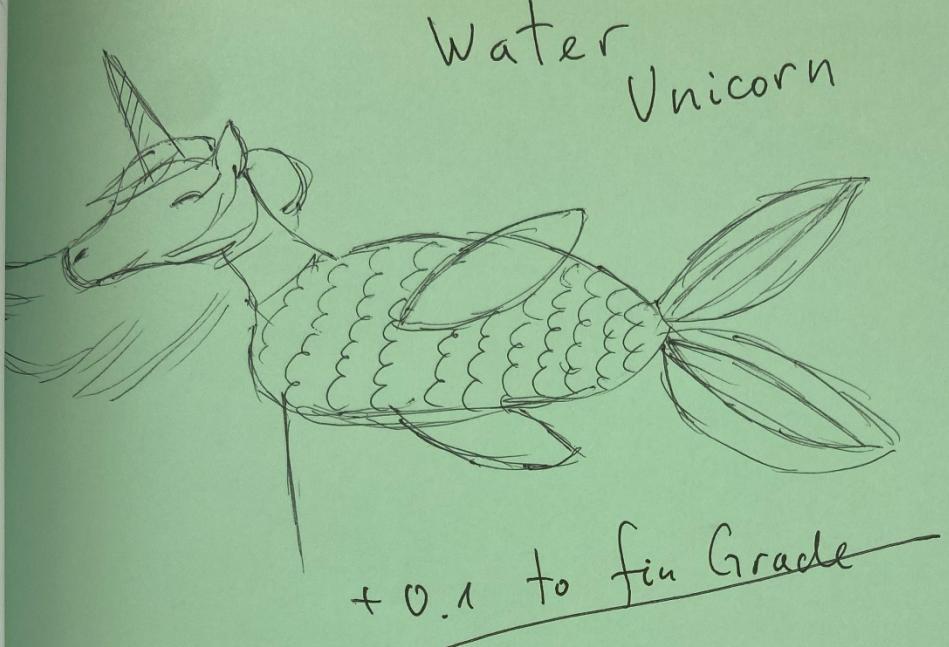
$\boxed{-1,45}$



White ①
 Black ②
 □ ③

$\square \blacksquare \square \blacksquare$
 $\square \blacksquare \square \blacksquare$
 $\square \blacksquare \square \blacksquare$
 $\square \blacksquare \square \blacksquare$

2 compoza



Water Unicorn

+ 0.1 to fin Grade

~~□ □ □~~ ~~3/8~~ Avg. num. of ?

$$\boxed{\text{■} \quad \text{■}} \quad \frac{3}{8} \quad 0,375 \quad 3+2+2+1=8$$

$$\boxed{\text{■} \quad \text{■}} \quad \frac{3}{8} \quad 0,375$$

$$\boxed{\text{■} \quad \text{■}} \quad \frac{3}{8} \quad 0,375 \quad \log_2 8 = 3$$

$$\boxed{\text{□}} \quad \frac{1}{8} \quad 0,125$$

$$\frac{3}{8} \cdot 1$$

(+)

$$\frac{1}{8} \cdot 2$$

$$\frac{1}{8} \cdot 2$$

$$\frac{1}{8} \cdot 3$$

$$\sum_{i=1}^N p_i \log_2 \left(\frac{1}{p_i} \right) \Rightarrow$$

$$\frac{5}{8} + \frac{2}{8} \quad \frac{3}{8} + \frac{3}{8} = \frac{13}{8} = 1,625$$

	①	②
1	++	++ ++ +
2	↑ ↓	↑ ↓
3	++	++

$$\frac{5}{8} \cdot 1,625$$

$$\frac{5}{8} \cdot 1 + \frac{1}{8} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3$$

$$= 1,375$$

$$1,375$$

$$\frac{2}{8} \cdot 4 = \frac{1}{1}$$

Wrong

Q 1. Красиво (yes)
Q 2. Чупба (No)

↑ System

2 Questions

$$\boxed{\log_2(4) = 2}$$

$\log_2^{1/2}$ Nightist
formula $0,625$

$$\begin{array}{r} 815 \\ \times 1,625 \\ \hline 50 \\ \hline 0,625 \end{array}$$

$$0,625 \log_2 \left(\frac{8}{5} \right) + 0,25 \cdot \log_2 \frac{8}{1}$$

$$0,5 - 0,75 + \underbrace{0,25 \cdot 3 \cdot 3}_{1,5}$$

$$\begin{array}{r} 5 \\ \times 1,5 \\ \hline 0,452 \\ \hline 1,952 \end{array}$$

$$\begin{aligned} & \frac{5}{8} + \frac{2}{8} + \frac{3}{8} + \frac{3}{8} = \frac{13}{8} = 1,625 \\ & 1,375 \quad 1,625 \end{aligned}$$

$$\begin{array}{r} \text{X} \text{ Y} \\ 1 \cdot 0,25 \cdot 2 \cdot 0,25 \\ 0,25 \quad 0,5 \end{array} \quad \begin{array}{r} 3 \cdot 0,25 + 3 \cdot 0,25 \\ 0,25 \quad 0,75 \end{array}$$

$$\boxed{2,25}$$

$$\begin{aligned} & \textcircled{1} \quad \sum_{i=1}^N p_i \log_2 \left(\frac{1}{p_i} \right) = \\ & \quad \left(0,25 \cdot \log_2 \frac{1}{0,25} \right) \cdot 4 = 2 \end{aligned}$$

$$\begin{aligned} & \textcircled{2} \quad \sum_{i=1}^N p_i \log_2 \left(\frac{1}{p_i} \right) = 0,625 \cdot \log_2 \frac{1}{0,625} \\ & \quad + 0,25 \cdot \log_2 \frac{1}{0,25} + \left(0,375 \cdot \log_2 \frac{1}{0,375} \right) \cdot 2 \\ & \approx 0,42 + 1,56 \approx 1,98 \end{aligned}$$

$$\begin{array}{r} 5 \\ \times 1,5 \\ \hline 0,452 \\ \hline 1,952 \end{array}$$